Closing Thurs: 1.6
Closing Tues: 2.1
Closing Next Thurs: 2.2, 2.3(part 1)
Midterm 1 will be returned Tuesday.

On the notecard put:

1. Your name (first & last)
2. Your quiz section
3. One questions about the current material or the course.

Remember to hand these in by the end of class for attendance credit.

2.1 Quadratics

Entry Task: An algebra warm up

1. Solve
   (a) $w^2 = 25$
   (b) $3q^2 - 1 = 11$
   (c) $(z - 1)^2 = 7$

2. Expand out these expression:
   (a) $(x - 3)(x - 4) =$
   (b) $(x + 5)(x - 6) =$
   (c) $(x - 7)^2 =$
   (d) $(A + B)^2 =$

3. For this class, you can get away without knowing how to factor, but many problems are easier and faster if you can factor.
   Try these (fill in the numbers):
   $x^2 - 8x + 16 = (x - \quad)(x - \quad)$

   $x^2 + 9x + 20 = (x + \quad)(x + \quad)$

A **quadratic function** can be written in the form:

$$y = ax^2 + bx + c.$$

The graph of a quadratic function is called a *parabola*.

For example:

$$y = -5x^2 + 20x + 30.$$

a = , b = , c =

$$f(x) = 4 + 2x^2.$$

a = , b = , c =

$$P(q) = (10q - 5q^2) - (3q + 6).$$

a = , b = , c =

A brief motivation:

1. If x = quantity, and
   $p = 105 - 0.1x$ = price (demand)
   then
   $TR(x) = (105 - 0.1x)x$ (why?)
   $= 105x - 0.1x^2$ (why?)
   a = , b = , c =

   It's fairly common for TR, Profit and/or AVC to be quadratic.

2. Projectiles. Throw an object in air.
   Example: A ball is thrown in the air from an initial height of 6 ft with an initial upward velocity of 20 ft/s.
   The height in feet is given by
   $$h(t) = 6 + 20t - 32t^2$$
   a = , b = , c =

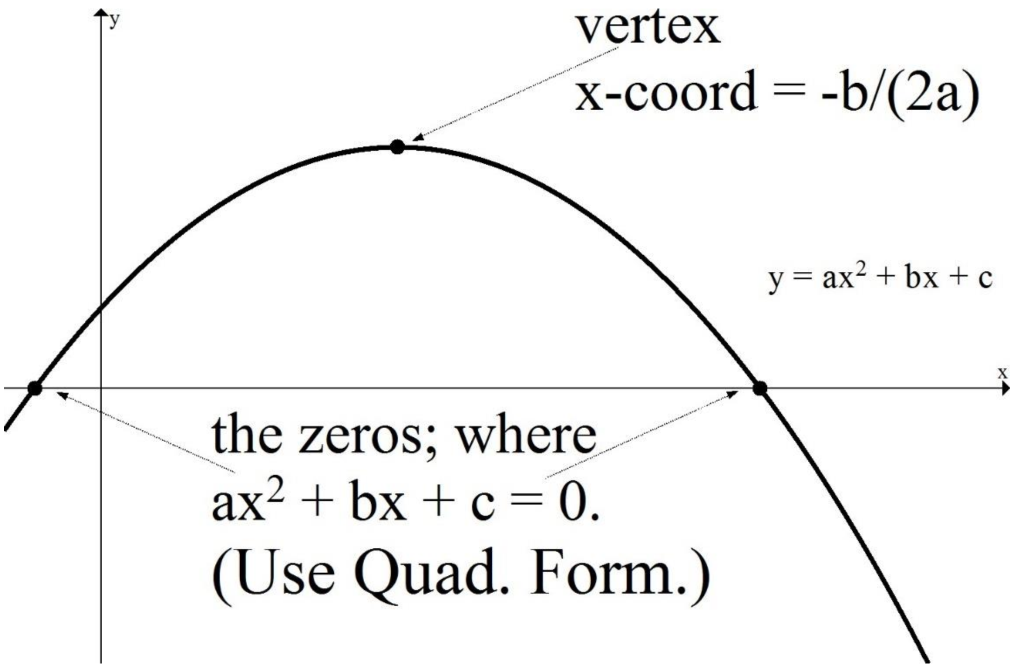
Parabola Basics

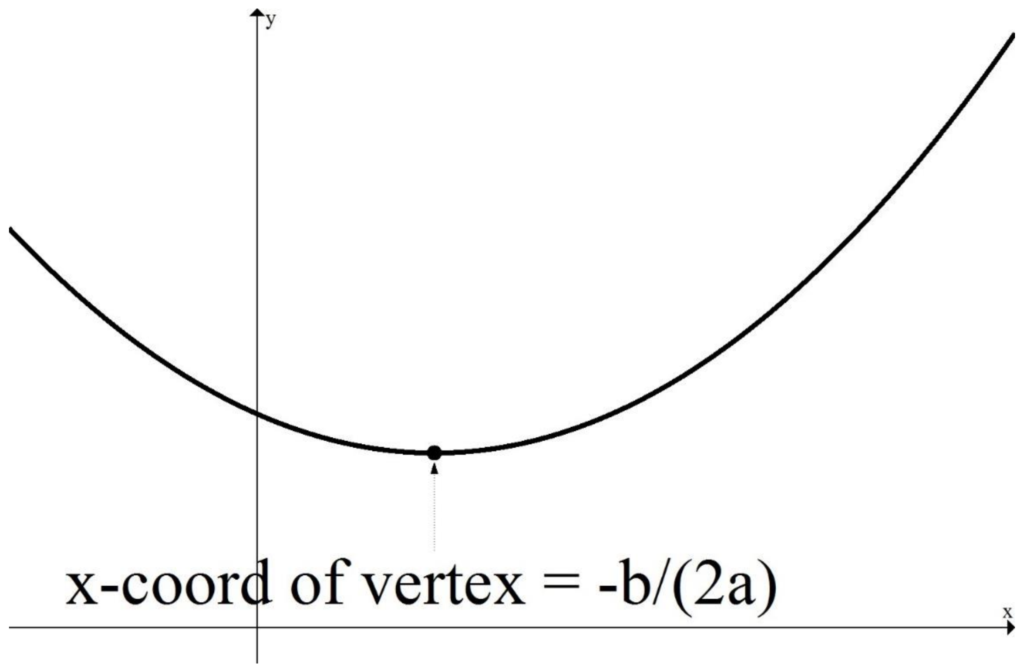
If *a* is negative, the parabola opens downward. For example:

$$y = -5x^2 + 20x + 30.$$

If *a* is positive, the parabola opens upward. For example:

$$y = 2x^2 + 28x + 4.$$





Note: A **quadratic equation** is an equation that can be written in the form

$$ax^2 + bx + c = 0.$$

The solution(s) to $ax^2 + bx + c = 0$ are given by the *quadratic formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Faster to solve, *if you can factor*:

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$
$$x^2 + 6x - 7 = 0$$
$$(x - 1)(x + 7) = 0$$

Thus,

$$x - 1 = 0 \quad \text{or} \quad x + 7 = 0$$
$$x = 1 \quad \text{or} \quad x = -7$$

A <u>general</u> solving method:

(Doesn't have to memorize a formula)

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$

Completing the square gives

$$x^2 + 6x + 9 - 9 = 7$$
$$(x + 3)^2 - 9 = 7$$
$$(x + 3)^2 = 16$$
$$x + 3 = 4 \quad \text{or} \quad x + 3 = -4$$
$$x = 1 \quad \text{or} \quad x = -7$$

The following will always work as well.

Solving with the quadratic formula:

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$
$$x^2 + 6x - 7 = 0$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-7)}}{\big(2(1)\big)}$$
$$= \frac{-6 \pm \sqrt{26 + 28}}{2}$$
$$= \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2}$$

Thus,

$$x = \frac{-6 + 8}{2} = 1 \text{ or } x = \frac{-6 - 8}{2} = -7$$

You can use any of these 3 methods!
All give the same answer.

For your own interest

(**Don't need to write this down**)

Derivation of the quadratic formula:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Completing the square:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Method for solving quadratic equations:

Given a *quadratic* ***equation***

(an *equation* has only one variable, an equal sign, *there is no y or f(x)*)

1. Simplify/Clear denominators.
2. Subtract to make one side zero. You will have something like:
$$ax^2 + bx + c = 0$$
3. Use the quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note (looking under the radical):

If $b^2 - 4ac > 0$, then two solutions.

If $b^2 - 4ac = 0$, then one solution.

If $b^2 - 4ac < 0$, then no solutions.

Solve:

1. $x^2 - 7x = 0$

2. $7 + 2x - 2x^2 = 4 + x$

3. $\frac{x}{3} - 4x^2 = 2x - 1$

Method for finding the vertex:

Given a *quadratic* ***function*** (there are two variables, input/output, and you want to find the vertex):

Given $$y = ax^2 + bx + c$$

or

$$f(x) = ax^2 + bx + c$$

The *x-coordinate of the vertex* is at:

$$x = -\frac{b}{2a}$$

Find the *x* and *y* coordinates of the vertex for (note that these are NOT equations, they are functions):

1. $y = 30 - 5x^2 + 20x.$

2. $y = 42x - x^2$

Example: (A preview of next week)
This is what you are doing in the first 6 questions in the 2.3 homework.

Suppose total revenue (TR) and total cost (TC) are given by

$R(x) = 42x - x^2$ and $C(x) = 50 + 3x$

where x is in hundred items and
$R(x)$, $C(x)$ are in hundred dollars.

(a) At what quantity is TR maximum?
(b) What is the maximum TR?
(c) Find the break-even points (*i.e.* quantities where profit is zero).
This is <u>not</u> the same as breakeven <u>price</u>!
(d) What quantity maximizes profit?